

The protophobic X -boson unified to the quantum electrodynamics

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The possible being of a X -boson with mass around the 17 MeV motivates us to construct its unification with the Quantum Electrodynamics (QED). This hypothetical particle would be a candidate to describe a Fifth Interaction, in a new physics beyond the Standard Model. The unification of the X -boson with the QED is based on composite symmetry $U(1)_{em} \times U(1)_X$, in which the group $U(1)_X$ is attached to the X -boson. The Higgs sector is constructed to generate a mass for the new boson, and so the mass of 17 MeV fixes the vacuum expected value scale. Thereby, we can estimate the mass of the hidden Higgs field through the VEV-scale and the weaker coupling constants. The model of quantum field theory is so constructed in a renormalizable gauge. Posteriorly, the radiative correction in the propagator of the X -boson is calculated at the one loop approximation. Furthermore, we also calculate the form factors associated with the QED-vertex correction. The precision of the electron's anomalous magnetic moment is useful to estimate the interaction magnitude of the X -boson with the fermions of the Standard Model.

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I. INTRODUCTION

The anomalies of the excited state of 8-Beryllium (8Be^*) to its ground state has revealed the being of a new neutral X -boson through the nuclear decay $8\text{Be}^* \rightarrow 8\text{Be} + X$ [1]. Immediately, the X -boson decays into the electron-positron pair $X \rightarrow e^+ + e^-$. It has a vector nature like the photon, but it must have a mass of approximately $m_X = 17\text{MeV}$. In principle, its unification is associated with the introduction of an extra gauge symmetry $U(1)_X$, beyond the known gauge symmetry of the Standard Model (SM). Certainly, the being of new boson may lead to the emergence of a Fifth fundamental interaction in the Nature [2]. In this context, the extended SM is so based on the gauge symmetry $SU_c(3) \times SU_L(2) \times U_R(1) \times U(1)_X$. For a complete review on anomaly in Beryllium decays, see [3]. Recently, a huge number of references show the alternative models to describe this extended SM [4–6].

The effective Lagrangian that could describe this model is :

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} X_{\mu\nu}^2 - \frac{\chi}{2} X_{\mu\nu} F^{\mu\nu} \\ & + \frac{1}{2} m_X^2 X_\mu^2 + J_\mu X^\mu, \end{aligned} \quad (1)$$

in which J^μ is the current coupled to X^μ

$$J^\mu = \sum_{\Psi=e,u,d,\dots} e \chi_\Psi \bar{\Psi} \gamma^\mu \Psi, \quad (2)$$

and Ψ sets any fermion of the SM, *i. e.*, $\Psi = \{e, u, d, \dots\}$. The X -boson can also interact chirally with the SM

leptons via axial current [7]. The current (2) defines the characteristic of the so called protophobic interaction in which a weaker coupling constant driven by the magnitude of the χ_Ψ -parameter. We list some estimates of χ_Ψ for fermions of SM, following the X -boson phenomenology in the literature :

$$\begin{aligned} 6.1 \times 10^{-5} & < \chi_e < 4.2 \times 10^{-4} \\ 2.0 \times 10^{-4} & < \chi_u < 10^{-3} \\ 4.0 \times 10^{-4} & < \chi_d < 2.0 \times 10^{-3}, \end{aligned} \quad (3)$$

while that for the neutrino, the experimental constraints indicate $\chi_\nu \simeq 0$.

In (1), the χ -parameter mixes kinetically the boson X^μ with the usual electromagnetic (EM) photon A^μ , where $X^{\mu\nu}$ is the field strength tensor of X^μ , and $F^{\mu\nu}$, the correspondent one of the photon. It is clear that the massive term spoils the symmetry $U(1)_X$, and the Lagrangian has just the EM gauge symmetry $U_{em}(1)$. Thereby, the idea is that the introduction of a spontaneous symmetry breaking (SSB) mechanism spoils the Abelian symmetry $U(1)_X$ to generate the mass $m_X = 17\text{MeV}$ in (1). Consequently, the experimental value of 17 MeV defines the scale of a vacuum expected value (VEV), and we can estimate a range of mass for the hidden Higgs scalar field.

In this paper, we construct an Abelian model based on $U(1)_{em} \times U(1)_X$ to describe the unification of the X -boson with the EM-photon. Thus, we have a scenario of quantum electrodynamics (QED) unified with the hypothetical Fifth interaction coming from the X -boson. The hidden scalar field is so introduced in the model to spoil the gauge symmetry $U(1)_X$, and consequently, the mass term for the X -boson emerges from the VEV-scale. Thus, we get a renormalizable and unitary model, in which the interaction between the X -boson and fermions of the SM satisfies the protophobic condition mentioned in (1). Thereby, we study some aspects of the model in

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view of a quantum field theory. For example, we estimate the time decay of the X -boson by using the decay rate of $X \rightarrow e^+ + e^-$. We obtain the potential correspondent to the scattering electron-positron into the X -boson, at the tree level. Posteriorly, we study some aspects of the perturbation theory at the one loop approximation : (i) The contribution of the X -boson to the electron physical mass; (ii) The X -boson full propagator and the m_X -renormalized mass; (iii) The contribution of the X -boson to the QED-vertex. Thus, the electron's anomalous magnetic moment gives a considerable estimation for the χ_e -protophobic parameter.

The organization of this paper follows the schedule: in section II we construct the model based on the symmetry $U(1)_{em} \times U(1)_K$. In section III, we propose the hidden Higgs sector to give the mass for the X -boson. In section IV, we calculate the decay rate of the X -boson into the electron-positron pair. In section V, the potential for the scattering electron-positron into the X -boson is obtained. In section VI, the contribution of the X -boson to the electron's self-energy is calculated at the one-loop. In section VII, the correction to the X -boson full propagator is used to obtain the correspondent Uehling Potential. In section VIII, the correction to the QED-vertex is calculated due to protophobic X -boson interaction with the electron-positron pair. Finally, the conclusions were depicted in section IX.

II. SETTING UP THE ABELIAN MODEL

In this section, we construct the sector of gauge fields and leptons of the model governed by the symmetry $U_J(1) \times U_K(1)$. We start with the sector of gauge fields set by two massless vector fields $B^\mu - Y^\mu$

$$\mathcal{L}_{gauge} = -\frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} Y_{\mu\nu}^2 - \frac{\chi}{2} B_{\mu\nu} Y^{\mu\nu}, \quad (4)$$

where χ is a real parameter that mixes the abelian gauge fields of the model, and as usual, we define the field strength tensors $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$. It is simple to verify that (4) is invariant under the gauge transformation

$$\begin{aligned} B_\mu &\mapsto B'_\mu = B_\mu - \partial_\mu f, \\ Y_\mu &\mapsto Y'_\mu = Y_\mu - \partial_\mu h, \end{aligned} \quad (5)$$

where f and h are arbitrary functions of the space-time.

The sector of leptons is defined by the usual Lagrangian

$$\mathcal{L}_{leptons} = \bar{\ell}(x) (i \not{D} - m_\ell) \ell(x), \quad (6)$$

where we have introduced the covariant derivative that couples the leptons family $\ell = (e, \mu, \tau)$ to gauge fields $B^\mu - Y^\mu$ defined in the following way :

$$D_\mu \ell = \left(\partial_\mu + i J g B_\mu + i Y \chi_y g' Y_\mu \right) \ell. \quad (7)$$

Here we have used J as the charge generator of $U_J(1)$, Y as that one of $U_Y(1)$, g and g' are coupling constants. The real parameter χ_y was introduced to fix the protophobic interaction of Y^μ - boson with leptons of the SM. Concerning the sectors $U_J(1)$ and $U_Y(1)$, the leptons are transformed such as

$$\begin{aligned} \ell &\mapsto \ell'(x) = e^{i J g f(x)} \ell(x), \\ \ell &\mapsto \ell'(x) = e^{i Y \chi_y g' h(x)} \ell(x), \end{aligned} \quad (8)$$

so we obtain the same transformations for the covariant derivatives. The set of all previous transformations defines the action of the model as being $U_J(1) \times U_Y(1)$ invariant. More explicitly, the interactions from (6) between leptons of the SM and the bosons $B^\mu - Y^\mu$ are given by

$$\mathcal{L}_{lept-gauge}^{int} = -\bar{\ell} (J g \not{B} + Y \chi_y g' \not{Y}) \ell. \quad (9)$$

In fact, we did not identify the real photon and X -boson in the model yet. In the next section, the Higgs mechanism must be introduced to generate the mass of new X -boson, and after the SSB, the massless gauge field remaining will be identified as the usual EM photon.

III. THE HIGGS SECTOR AND THE MASS OF X-BOSON

We have constructed a model of a Abelian gauge fields to describe a possible scenario of unification between QED and the protophobic X -boson interaction. In this section, we propose a Higgs sector that breaks one of Abelian symmetries to give mass to the new X -boson. After this spontaneous breaking symmetry (SSB), we will obtain the EM symmetry such as $U_J(1) \times U_Y(1) \xrightarrow{\langle \Phi \rangle} U_{em}(1)$. Then we will denote this Higgs field as the scalar field Φ . To do that, we will introduce the Higgs sector Lagrangian

$$\mathcal{L}_{Higgs} = |D_\mu \Phi|^2 - \mu^2 |\Phi|^2 - \lambda |\Phi|^4, \quad (10)$$

where μ and λ are real parameters. The covariant derivative of (10) acts on the Higgs- Φ just coupling to the Abelian gauge fields

$$D_\mu \Phi = \left(\partial_\mu + i J_\Phi g B_\mu + i Y_\Phi \chi_y g' Y_\mu \right) \Phi. \quad (11)$$

The complex field Φ is a scalar singlet that has the transformation under symmetry group $U_J(1) \times U_Y(1)$

$$\begin{aligned} \Phi &\mapsto \Phi'(x) = e^{i J_\Phi g f(x)} \Phi(x), \\ \Phi &\mapsto \Phi'(x) = e^{i Y_\Phi \chi_y g' h(x)} \Phi(x). \end{aligned} \quad (12)$$

Using the previous gauge transformations, it is immediate that $D_\mu \Phi$ has the same transformation of (12), thus the Lagrangian (10) is invariant under gauge symmetry (12).

The minimal value of the Higgs potential is obtained by the non-trivial vacuum expected value (VEV) of the Higgs field that keeps the full invariance of the model. We choose it as the VEV constant $\langle \Phi \rangle_0 = v/\sqrt{2}$, in which v is the non-trivial VEV of the scalar field Φ , defined by $v := \sqrt{-\frac{\mu^2}{\lambda}}$, when $\mu^2 < 0$. We choose the parametrization of the Φ -complex field as

$$\Phi(x) = \left(\frac{v + H(x)}{\sqrt{2}} \right) e^{i \frac{\eta(x)}{v}}, \quad (13)$$

where H and η are real functions. The VEV- v defines a scale for the break of the composite Abelian symmetry, in which one of the Abelian gauge fields acquires a mass term. Thus, after SSB, the Abelian sector is given by

$$\begin{aligned} \mathcal{L}_{gauge}^{B-Y} = & -\frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} Y_{\mu\nu}^2 - \frac{\chi}{2} B_{\mu\nu} Y^{\mu\nu} \\ & + \frac{v^2}{2} \left(J_\Phi g B_\mu + Y_\Phi \chi_y g' Y_\mu \right)^2 + \frac{1}{2} (\partial_\mu \eta)^2 \\ & + v \partial_\mu \eta \left(J_\Phi g B^\mu + Y_\Phi \chi_y g' Y^\mu \right). \end{aligned} \quad (14)$$

The sector $B^\mu - Y^\mu$ suggests us to introduce the shift to eliminate the χ -mixing term

$$\begin{aligned} B^\mu &= A^\mu - \frac{\chi}{\sqrt{1-\chi^2}} X^\mu \\ Y^\mu &= \frac{X^\mu}{\sqrt{1-\chi^2}}, \end{aligned} \quad (15)$$

thus the gauge sector can be written as

$$\begin{aligned} \mathcal{L}_{gauge}^{A-X} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} X_{\mu\nu}^2 + \frac{g^2 v^2}{2} \left(J_\Phi A_\mu + \chi_\Phi X_\mu \right)^2 \\ & + \frac{1}{2} (\partial_\mu \eta)^2 + g v \partial_\mu \eta \left(J_\Phi A_\mu + \chi_\Phi X_\mu \right). \end{aligned} \quad (16)$$

We have defined χ_Φ as

$$\chi_\Phi := -J_\Phi \tilde{\chi} + Y_\Phi \tilde{\chi}_y \frac{g}{g'}, \quad (17)$$

where $\tilde{\chi}$ and $\tilde{\chi}_y$ are, respectively, given by

$$\tilde{\chi} := \frac{\chi}{\sqrt{1-\chi^2}} \quad \text{and} \quad \tilde{\chi}_y := \frac{\chi_y}{\sqrt{1-\chi^2}}. \quad (18)$$

Since we identify A^μ as the EM photon, so we choose $g = g' = e$, and consequently, $Q_{em}^{(\Phi)} = J_\Phi = 0$. The Higgs charge is chosen in the literature by the value of $Y_\Phi = 3$, so the χ_Φ -parameter is $\chi_\Phi = 3\tilde{\chi}_y$. In particular, this value for the Higgs charge is associated with the instability of others fermions that can be introduced in the model, and it would be bounded to a Dark matter scenario, for more details, see [8–10]. Thus the previous Lagrangian is reduced to

$$\begin{aligned} \mathcal{L}_{gauge}^{X-A} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} X_{\mu\nu}^2 + \frac{1}{2} m_X^2 X_\mu^2 \\ & + \frac{1}{2} (\partial_\mu \eta)^2 + \frac{1}{2} m_X \partial_\mu \eta X^\mu, \end{aligned} \quad (19)$$

in which the mass of X -boson is identified in terms of VEV scale- v as

$$m_X = 3e |\tilde{\chi}_y| v. \quad (20)$$

To eliminate the mixed term $\eta - X^\mu$ in (19), we add the gauge fixing Lagrangian

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha} (\partial_\mu A^\mu)^2 - \frac{1}{2\beta} \left(\partial_\mu X^\mu - \frac{\beta}{2} m_X \eta \right)^2, \quad (21)$$

where $\{\alpha, \beta\}$ are real parameters. The surface terms are eliminated if we imagine the Lagrangian integrated by throughout the space-time, so we meet all the terms of the gauge sector

$$\begin{aligned} \mathcal{L}_{gauge-0} = & -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 \\ & -\frac{1}{4} X_{\mu\nu}^2 - \frac{1}{2\beta} (\partial_\mu X^\mu)^2 + \frac{1}{2} m_X^2 X_\mu^2. \end{aligned} \quad (22)$$

The X -mass defines a massive neutral boson associated with the scale of v VEV-scale. We will identify the X^μ gauge field as the hypothetical particle suggested by the Beryllium nuclear decay, and A^μ is interpreted as the EM-photon. Then $F^{\mu\nu}$ is the strength field tensor of A^μ , as usual, we have $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. The Higgs mass after the VEV is given by $m_H = \sqrt{2\lambda} v^2$. All propagators into the quadratic sector have well behaviour at the ultraviolet range, *i. e.*, it is renormalizable. The inversion of quadratic Lagrangian (22) gives the propagators in the momentum space

$$\begin{aligned} \langle A_\mu A_\nu \rangle &= -\frac{i}{k^2} \left[\eta_{\mu\nu} + (\alpha - 1) \frac{k_\mu k_\nu}{k^2} \right] \\ \langle X_\mu X_\nu \rangle &= -\frac{i}{k^2 - m_X^2} \left[\eta_{\mu\nu} + (\beta - 1) \frac{k_\mu k_\nu}{k^2 - \beta m_X^2} \right]. \end{aligned} \quad (23)$$

To clarify the emergence of the EM interaction, as well as, the protophobic interaction of leptons with the X -boson, we back to interactions as highlighted in (9). It is written in terms of the fields X^μ and A^μ as

$$\mathcal{L}_{lept-gauge}^{int} = -e Q_{em} \bar{\ell} \not{A} \ell - e \chi_\ell \bar{\ell} \not{X} \ell, \quad (24)$$

in which the electric charge generator is identified as $Q_{em} = J = -1$ for leptons family, and the milli-charged generator is defined by

$$\chi_\ell := -Q_{em} \tilde{\chi} + Y \tilde{\chi}_y. \quad (25)$$

In the case of leptons family the χ -parameter has the generators $Q_{em} = -1$ and $Y = 0$, so we obtain the relation

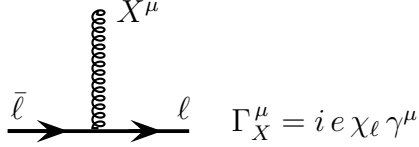
$$\chi_e = \chi_\mu = \chi_\tau = \tilde{\chi}. \quad (26)$$

Since we use the constraint associated with Magnetic moment of the electron $(g-2)_e$ [11], so it allows to fix the upper limit $|\chi_e| < 1.4 \times 10^{-3}$. To estimate the $\tilde{\chi}_y$ -parameter, we analyze the charges in the scenario of EM

interaction with up- and down-quarks. In this case, the χ_ℓ -parameter in (25) for up and down-quarks, respectively, are given by

$$\begin{aligned}\chi_u &= -\frac{2}{3}\tilde{\chi} + \frac{1}{3}\tilde{\chi}_y \\ \chi_d &= +\frac{1}{3}\tilde{\chi} + \frac{1}{3}\tilde{\chi}_y,\end{aligned}\quad (27)$$

where $Y = +1/3$ has been chosen by convenience. Using the relation for nucleon charges $\chi_n = \chi_u + 2\chi_d$, we obtain that $\chi_n = \tilde{\chi}_y = 3.0 \times 10^{-3}$. Thus the χ -parameters that emerge in the model are constrained by experimental estimative. The vertex correspondent to interaction X -boson-lepton in (24) is given by:



$$\Gamma_X^\mu = i e \chi_\ell \gamma^\mu$$

If we use the value $m_X = 17$ MeV, the X^μ -boson mass determines the VEV-scale as

$$v \simeq \frac{19}{|\tilde{\chi}_y|} \text{ MeV}, \quad (28)$$

in which it fixes the VEV scale at $v \simeq 6.3$ GeV. Thereby, using the range of $0.1 < \lambda < 0.9$ for the coupling constant from (10), the Dark Higgs mass is so estimated in the range

$$2.8 \text{ GeV} < m_H < 8.5 \text{ GeV}. \quad (29)$$

For end, we get the final gauge symmetry, after this SSB, through the gauge transformation in the basis of the fields $\{A^\mu, X^\mu\}$. Using the inverse transformation of (15), we obtain

$$\begin{aligned}A_\mu &\mapsto A'_\mu = A_\mu - \partial_\mu (f + \chi h), \\ X_\mu &\mapsto X'_\mu = X_\mu - \partial_\mu \left(\frac{\tilde{\chi}}{\chi} h \right),\end{aligned}\quad (30)$$

After the SSB, we fix the condition $h = 0$, so the final EM gauge symmetry is

$$\begin{aligned}A_\mu &\mapsto A'_\mu = A_\mu - \partial_\mu f(x), \\ X_\mu &\mapsto X'_\mu = X_\mu.\end{aligned}\quad (31)$$

Therefore we have obtained a model of quantum field theory consistent with the requirement of renormalization and unitarity. Furthermore, the interaction sector of X -boson with the leptons/quarks of the SM satisfies all the experimental constraints of protophobic interaction. In the next section, we will analyze the decay rate to estimate the time decay of the X -boson.

IV. THE TIME DECAY OF THE X -BOSON

The X -boson decay into the leptons pair of the SM is represented by the vertex :

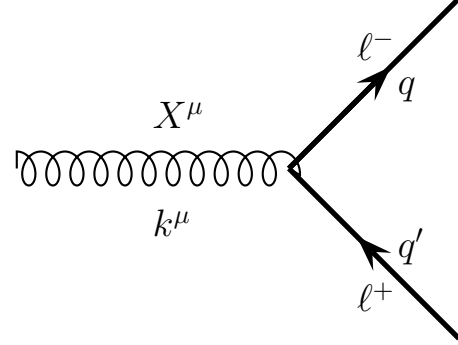


FIG. 1: VERTEX DIAGRAM FOR THE X -BOSON DECAY WIDTH.

Using the usual rules of QFT, the decay rate is given by the expression

$$\Gamma = \frac{1}{2\pi^2} \frac{1}{2k^0} \int \frac{d^3q}{2q^0} \int \frac{d^3q'}{2q'^0} \delta^4(k - q - q') \frac{1}{12} \sum_{\lambda, s, s'} |\mathcal{M}|^2, \quad (32)$$

where k^μ is the X -boson four-momentum, and q and q' are the external momenta of the electron and positron, respectively. The electron-positron elastic scattering amplitude is

$$\mathcal{M} = e \chi_\ell \epsilon_\mu(k, \lambda) \bar{u}_\ell(q, s) \gamma^\mu v_{\ell'}(q', s'), \quad (33)$$

where ϵ^μ is the polarization vector, u and v represent the fermions amplitude functions. Using the completeness relation

$$\begin{aligned}\sum_\lambda \epsilon_\mu(k, \lambda) \epsilon_\nu(k, \lambda) &= -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_X^2}, \\ \sum_s u_\ell(q, s) \bar{u}_{\ell'}(q, s) &= (\not{q} + m)_{\ell\ell'}, \\ \sum_{s'} v_{\ell'}(q', s') \bar{v}_{\ell'}(q', s') &= (\not{q}' - m)_{\ell'\ell'},\end{aligned}\quad (34)$$

the decay factor Γ assumes the form

$$\begin{aligned}\Gamma &= \frac{e^2 \chi_\ell^2}{12\pi^2 m_X^3} \int \frac{d^3q}{2q^0} \int \frac{d^3q'}{2q'^0} \delta^4(k - q - q') \\ &\times \left[(q \cdot k)(q' \cdot k) + \frac{1}{4} m_X^2 (m_X^2 + 4m_\ell^2) \right],\end{aligned}\quad (35)$$

where we have used the on-shell condition $k^2 = m_X^2$. Solving the above integral, we arrive at the decay rate:

$$\Gamma(X \rightarrow \ell^+ \ell^-) = \frac{e^2 \chi_\ell^2}{48\pi} m_X \sqrt{1 - 4 \frac{m_\ell^2}{m_X^2}} \left(1 + 2 \frac{m_\ell^2}{m_X^2} \right). \quad (36)$$

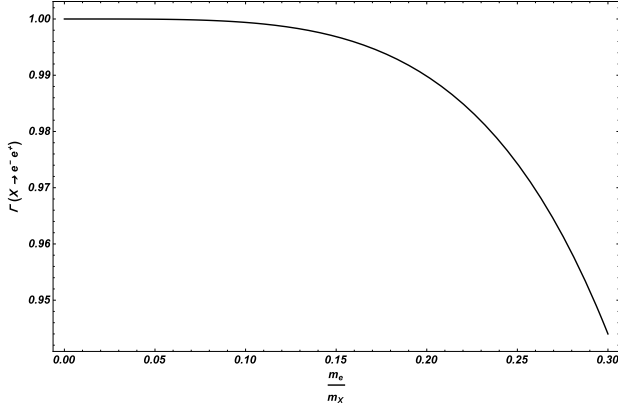


FIG. 2: The X -boson decay rate as function of the ratio m_e/m_X .

Since the decay factor satisfies the condition $\Gamma_T > 3\Gamma_\ell$ for each lepton, the decay $X^\mu \rightarrow e^+ + e^-$ can be approximated by $m_X \gg 2m_e$, so the Γ factor reduces to

$$\Gamma(X \rightarrow e^+ e^-) \approx \frac{e^2 \chi_e^2}{48\pi} m_X. \quad (37)$$

In general, there may be many decays modes, what means that the massive mode lifetime τ is given by

$$\tau = \frac{1}{\Gamma_T} < \frac{16\pi}{e^2 \chi_e^2 m_X}, \quad (38)$$

in which using the mass $m_X = 17 \text{ MeV}$, it gives a lifetime of $\tau \lesssim 10^{-14} \text{ s}$.

V. POTENTIAL OF X -BOSON FOR THE $e^+ - e^-$ SCATTERING

The potential function associated with the scattering electron-positron into the X -Boson is obtained here in this section by using the previous rules. The process is so represented by the diagram :

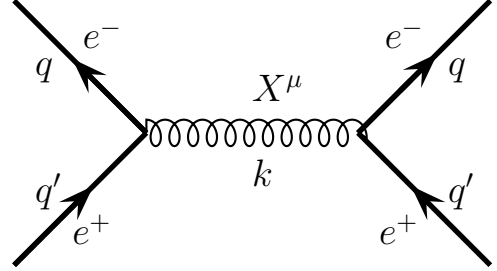


FIG. 3: THE SCATTERING PROCESS INVOLVING THE INTERACTION OF X -BOSON WITH THE ELECTRON-POSITRON PAIR AT THE TREE LEVEL.

Using the rules of the diagram above, the amplitude for this scattering at the tree level is

$$iA^{(XX)} = \bar{u}(q) (-ie\chi_e \gamma_\mu) u(q') \times \langle X^\mu X^\nu \rangle \bar{u}(-q') (-ie\chi_e \gamma_\nu) u(q). \quad (39)$$

where the momentum are $q = p + k/2$ and $q' = p - k/2$, so the X -boson momenta is $k = q - q'$. In the non-relativistic limit, the potential in the momentum space is identified as

$$\tilde{V}^{(XX)}(k^2) = \frac{\chi_e^2 e^2}{k^2 - m_X^2}. \quad (40)$$

In field theory, the potential correspondent to the scattering amplitude is defined by the Fourier integral

$$V(\vec{r}) = -\chi_e^2 e^2 \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{e^{i \vec{k} \cdot \vec{r}}}{\vec{k}^2 + m_X^2}, \quad (41)$$

in which the result gives us a Yukawa potential

$$V(r) = -\chi_e^2 e^2 \frac{e^{-m_X r}}{4\pi r}. \quad (42)$$

VI. THE ELECTRON SELF-ENERGY

Using the previous rules, the electron propagator at the one loop approximation can be written as the sum of usual QED contribution and the self-energy due to X -boson propagator

$$\Sigma_1(\not{p}) = \Sigma_1^{QED}(\not{p}) + \Sigma_1^{(X)}(\not{p}) . \quad (43)$$

These integrals have linear divergencies in the ultraviolet range, so we will use the dimensional regulator to control divergencies, *i. e.*, the integral dimension is altered by $4 \rightarrow 2\omega$, in which the physical dimension is recovered when $\omega \rightarrow 2$. For simplicity, we choose the Feynman's gauge, where the gauge fixing parameters are given by $\alpha = \beta = 1$. In this case, the QED contribution is known in the literature by the result

$$\begin{aligned} \Sigma_1^{QED}(\not{p}, \omega) &= \frac{e^2}{8\pi^2} \Gamma(2-\omega) \times \\ &\times \int_0^1 dz [(1-\omega)(1-z)\not{p} + \omega m] \\ &\times \left[\frac{4\pi\mu^2}{m^2 z - p^2 z(1-z)} \right]^{2-\omega}, \end{aligned} \quad (44)$$

where the condition $2m^2 > p^2$ must be satisfied to calculate the integral, and μ^2 is an arbitrary energy scale to turns out the coupling constant dimensionless in 2ω -dimension. The regularized contribution of X -boson is given by the integral

$$\begin{aligned} -i\Sigma_1^{(X)}(\not{p}, \omega) &= (ie\chi_e)^2 (\mu^2)^{2-\omega} \times \\ &\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{\gamma^\mu (\not{p} + \not{k} + m) \gamma_\mu}{[(p+k)^2 - m^2] (k^2 - m_X^2)}. \end{aligned} \quad (45)$$

Using the known technical in the literature for Feynman integrals, the condition $2(m_X^2 + m^2) > p^2$ gives us the integral result is

$$\begin{aligned} \Sigma_1^{(X)}(\not{p}, \omega) &= \frac{e^2 \chi_e^2}{8\pi^2} \Gamma(2-\omega) \times \\ &\times \int_0^1 dz [(1-\omega)(1-z)\not{p} + \omega m] \times \\ &\left[\frac{4\pi\mu^2}{m_X^2 - (m_X^2 - m^2)z - p^2 z(1-z)} \right]^{2-\omega}. \end{aligned} \quad (46)$$

The results have divergences at $\omega = 2$, and by analytic continuation of Gamma function it is well defined throughout the complex plane of ω , except at $\Re(\omega) = 2, 3, 4, \dots$, and so on. To return to the physical dimension, we take off the regularization parameter expanding

around the $\omega = 2 - \varepsilon$, with $\varepsilon \rightarrow 0$, so we obtain the result

$$\begin{aligned} \Sigma_1^{(X)}(\not{p}, \varepsilon) &\simeq \frac{e^2}{16\pi^2} (-\not{p} + 4m) \frac{1}{\varepsilon} \\ &- \frac{\gamma e^2}{16\pi^2} (-\not{p} + 4m) \\ &+ \frac{e^2}{8\pi^2} \int_0^1 dz [-(1-z)\not{p} + 2m] \\ &\times \ln \left[\frac{4\pi\mu^2}{m^2 z - p^2 z(1-z)} \right] \\ &+ \frac{e^2 \chi_e^2}{8\pi^2} \int_0^1 dz [-(1-z)\not{p} + 2m] \\ &\times \ln \left[\frac{4\pi\mu^2}{m_X^2 - (m_X^2 - m^2)z - p^2 z(1-z)} \right]. \end{aligned} \quad (47)$$

Here we isolate the divergent part summed to finite part of the the self-energy. The contribution to the electron's mass comes from the on-shell condition $\not{p} = m$, or $p^2 = m^2$. Imposing it on the result (47), we obtain that

$$\begin{aligned} \Sigma_1(\not{p} = m, \varepsilon) &\simeq \frac{3m\alpha}{4\pi} \frac{1}{\varepsilon} - \frac{3m\alpha\gamma}{4\pi} \\ &+ \frac{3m\alpha}{4\pi} \ln \left(\frac{4\pi\mu^2}{m^2} \right) + \frac{5m\alpha}{4\pi} \\ &+ \frac{7m\alpha\chi_e^2}{8\pi} - \frac{3m\alpha\chi_e^2}{2\pi} \ln \left(\frac{m_X}{m} \right), \end{aligned} \quad (48)$$

where we have assumed $m_X^2/m^2 \gg 1$, and $1 + \chi_e^2 \simeq 1$. This result gives the contribution at the one loop approximation for the electron's renormalized mass. Thereby, the electron's physical mass has the finite correction given by

$$m_e^{(R)} = m_e + \frac{5m_e\alpha}{4\pi} + \frac{7m_e\alpha\chi_e^2}{8\pi} - \frac{3m_e\alpha\chi_e^2}{2\pi} \ln \left(\frac{m_X}{m_e} \right). \quad (49)$$

VII. THE FULL X -PROPAGATOR AND THE UEHLING POTENTIAL

The renormalized field of X -boson is defined by the relation

$$X_0^\mu = \sqrt{Z_X} X^\mu, \quad (50)$$

in which the full renormalized propagator of the X -boson depends on the Z_X -factor, and it is given by the expression

$$\Delta_{\mu\nu}(k^2) = -\frac{i\eta_{\mu\nu} Z_X^{-1}}{(1 - \Pi(k^2)) k^2 - m_{0X}^2}. \quad (51)$$

The $\Pi(k^2)$ is a scalar function that multiplies the transverse term in the vacuum polarization

$$\Pi_{\mu\nu}(k^2) = \Pi(k^2) (\eta_{\mu\nu} k^2 - k_\mu k_\nu). \quad (52)$$

The conserved current guarantees that $k_\mu J^\mu = 0$, so the terms like $k_\mu k_\nu / k^2$ are null due to contraction $J_\mu \Delta^{\mu\nu} J_\nu$, in the perturbation theory. The on-shell condition $k^2 = m_X^2$ fixes the propagator pole by the Z_X -factor condition

$$Z_X = \frac{1}{1 - \Pi(k^2 = m_X^2)}, \quad (53)$$

thus, the X -propagator (51) is finite and given by

$$\Delta_{\mu\nu}(k^2) = -\frac{i \eta_{\mu\nu}}{[1 - \Pi_R(k^2)] k^2 - m_X^2}. \quad (54)$$

The scalar function $\Pi_R(k^2) = \Pi(k^2) - \Pi(k^2 = m_X^2)$ is finite, and it cancels the divergent coming from the vacuum polarization. We call attention here for the physical mass m_X , that is related to unphysical mass m_{0X} , in accord with the relation $m_{0X} = \sqrt{Z_X^{-1}} m_X$. Thereby, the renormalization factor for the X -boson mass is $Z_{m_X} = Z_X^{-1}$.

Using the previous rules, the expression of the vacuum polarization at the one-loop approximation is given by integral

$$i \Pi_1^{\mu\nu}(m, k) = -(i e \chi_e)^2 \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left(\frac{i \gamma^\mu}{\not{p} - m} \frac{i \gamma^\nu}{\not{p} - \not{k} - m} \right), \quad (55)$$

which is the same expression of quantum electrodynamics ordinary by the factor χ_e , and consequently, the calculus of vacuum polarization in this order is the same of usual QED. Using the dimensional regularization, *i. e.*, $D = 4 \rightarrow D = 2\omega$, the previous integral is calculated by methods known in textbooks, so we have the result

$$\begin{aligned} \Pi_1(m^2, k^2, \omega) &= -\frac{\alpha \chi_e^2}{\pi} (\mu^2)^{2-\omega} \omega \Gamma(2-\omega) \\ &\times \int_0^1 dx x(1-x) \left[\frac{4\pi\mu^2}{m^2 - k^2 x(1-x)} \right]^{2-\omega}, \end{aligned} \quad (56)$$

with $4m^2 > k^2$. We isolate the divergent part by writing $\omega = 2 - \varepsilon$, with $\varepsilon \rightarrow 0^+$ to get the result

$$\begin{aligned} \Pi_1(m^2, k^2, \varepsilon) &= -\frac{\alpha \chi_e^2}{3\pi} \frac{\mu^{2\varepsilon}}{\varepsilon} + \frac{\alpha \chi_e^2}{6\pi} (2\gamma + 1) \\ &- \frac{2\alpha \chi_e^2}{\pi} \int_0^1 dx x(1-x) \times \ln \left[\frac{4\pi\mu^2}{m^2 - k^2 x(1-x)} \right]. \end{aligned} \quad (57)$$

Using this result, the finite part that emerges in (54) is given by the subtraction

$$\Pi_R(k^2) = \frac{2\alpha \chi_e^2}{\pi} \int_0^1 dx x(1-x) \ln \left[\frac{m^2 - k^2 x(1-x)}{m^2 - m_X^2 x(1-x)} \right]. \quad (58)$$

The integral in (58) can be calculated for all range values

of k^2 , so we obtain

$$\begin{aligned} \Pi_R(k^2) &= -\frac{4\alpha \chi_e^2}{3\pi} \left(\frac{m^2}{k^2} - \frac{m^2}{m_X^2} \right) \\ &+ \frac{\alpha \chi_e^2}{3\pi} \left(1 + \frac{2m^2}{k^2} \right) f(k^2) \\ &- \frac{\alpha \chi_e^2}{3\pi} \left(1 + \frac{2m^2}{m_X^2} \right) f(k^2 = m_X^2), \end{aligned} \quad (59)$$

where the function $f(k^2)$ is defined by

$$f(k^2) = \begin{cases} 2\sqrt{1 - \frac{4m^2}{k^2}} \sinh^{-1} \left(\frac{\sqrt{-k^2}}{2m} \right) & \text{if } k^2 < 0, \\ \sqrt{\frac{4m^2}{k^2} - 1} \cot^{-1} \left(\sqrt{\frac{4m^2}{k^2} - 1} \right) & \text{if } 0 < k^2 \leq 4m^2, \\ \sqrt{1 - \frac{4m^2}{k^2}} \left[2 \cosh^{-1} \left(\frac{\sqrt{k^2}}{2m} \right) - i\pi \right] & \text{if } k^2 > 4m^2. \end{cases}$$

In this expression, we observe the emergence of an imaginary part, when $k^2 > 4m^2$. This is the X -boson case in that the on-shell condition $k^2 = m_X^2$ fixes the inequality $m_X^2 > 4m^2$, for the masses $m_X = 17 \text{ MeV}$ and $m_e = 0.5 \text{ MeV}$. The imaginary part means the instability of the X -boson, and as consequence, it decays into the virtual electron-positron pair.

The result of the integral (59) under the on-shell condition $k^2 = m_X^2$ yields the Z_X -factor

$$\begin{aligned} Z_X &\simeq 1 - \frac{\alpha \chi_e^2}{3\pi} \frac{\mu^{2\varepsilon}}{\varepsilon} + \frac{\alpha \chi_e^2}{6\pi} (2\gamma + 1) \\ &- \frac{\alpha \chi_e^2}{3\pi} \ln \left(\frac{4\pi\mu^2}{m^2} \right) - i 2\alpha \chi_e^2 \left(1 - \frac{2m^2}{m_X^2} \right) \\ &+ \frac{4\alpha \chi_e^2}{\pi} \left(1 - \frac{2m^2}{m_X^2} \right) \ln \left(\frac{m_X}{m} \right), \end{aligned} \quad (60)$$

thereby, the Z_{m_X} -factor is easily obtained, and the physical mass of the X -boson is

$$\begin{aligned} \frac{m_X^{(R)}}{m_{0X}} &\simeq 1 - \frac{\alpha \chi_e^2}{6\pi} \frac{\mu^{2\varepsilon}}{\varepsilon} + \frac{\alpha \chi_e^2}{12\pi} (2\gamma + 1) \\ &- \frac{\alpha \chi_e^2}{6\pi} \ln \left(\frac{4\pi\mu^2}{m^2} \right) - i \alpha \chi_e^2 \left(1 - \frac{2m^2}{m_X^2} \right) \\ &+ \frac{2\alpha \chi_e^2}{\pi} \left(1 - \frac{2m^2}{m_X^2} \right) \ln \left(\frac{m_X}{m} \right). \end{aligned} \quad (61)$$

The general expression to energy potential is given by

$$U(\mathbf{r}) = -4\pi\alpha\chi_e^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \Delta_{00}(-\mathbf{k}^2). \quad (62)$$

Applying the propagator (51) in the energy potential (62), we have neglected terms of the order α^3 , after some algebraic manipulation, we use the approximation $m^2/m_X^2 \ll 1$, and the previous integral is reduced to one

quadrature in the following expression :

$$\begin{aligned}
U(r) = & -\alpha \chi_e^2 \frac{e^{-m_X r}}{r} - i \frac{\alpha^2 \chi_e^4}{3r} \left(1 - \frac{m_X r}{2}\right) e^{-m_X r} \\
& - \frac{\alpha^2 \chi_e^4}{3\pi r} \left(1 + \frac{6m^2}{m_X^2}\right) e^{-m_X r} \\
& - \frac{2\alpha^2 \chi_e^4}{3\pi r} \int_1^\infty d\xi \left(1 + \frac{1}{2\xi^2}\right) \\
& \times \frac{(\xi^2 - 1)^{1/2}}{\xi^2} \left(1 - \frac{m^2}{4m_X^2 \xi^2}\right)^{-2} e^{-2mr\xi}. \quad (63)
\end{aligned}$$

This integral- ξ can be called the integral representation of the *Uehling potential* with the correction of the X-boson mass. The integral- ξ is difficult to solve analytically, so we analyse it for the asymptotic case $mr \gg 1$. For $mr \gg 1$, only the region $0 \leq \xi - 1 \ll (mr)^{-1}$ contributes for the integral, so one can approximate $\xi \simeq 1$ at some places, to obtain the expression

$$\begin{aligned}
U(r) \simeq & -\alpha \chi_e^2 \frac{e^{-m_X r}}{r} - i \frac{\alpha^2 \chi_e^4}{3r} \left(1 - \frac{m_X r}{2}\right) e^{-m_X r} \\
& - \frac{\alpha^2 \chi_e^4}{3\pi r} \left(1 + \frac{6m^2}{m_X^2}\right) e^{-m_X r} \\
& - \frac{\alpha^2 \chi_e^4}{4\sqrt{\pi} r} \left(1 + \frac{m^2}{2m_X^2}\right) \frac{e^{-2mr}}{(mr)^{3/2}}. \quad (64)
\end{aligned}$$

Obviously, the imaginary part goes to zero when $m_X \rightarrow \infty$, and the Uehling potential expression is recovered with the coupling constant $\alpha^2 \chi_e^4$.

VIII. THE ELECTRON ANOMALOUS MAGNETIC MOMENT

The QED-vertex has the contribution of the X-boson vertex at the one loop given by the expression :

$$\begin{aligned}
\Gamma_1^\mu(p, p') = & -i e^2 \chi_e^2 \int \frac{d^4 k}{(2\pi)^4} \times \\
& \frac{\gamma^\alpha (\not{k} + \not{p}' + m) \gamma^\mu (\not{k} + \not{p} + m) \gamma_\alpha}{[(k + p')^2 - m^2][(k + p)^2 - m^2](k^2 - m_X^2)}. \quad (65)
\end{aligned}$$

The correspondent diagram is illustrated below :

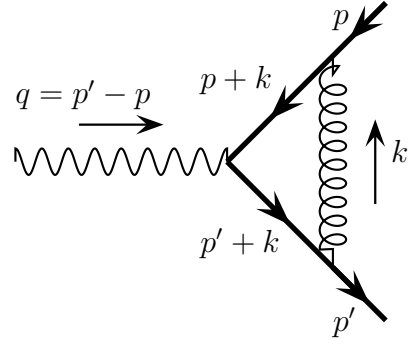


FIG. 4: THE ONE-LOOP CORRECTION OF THE X-BOSON TO THE QED VERTEX DIAGRAM.

This integral has a ultraviolet divergence by a simple power counting. Therefore, we use the previous technical of dimensional regularization to isolate the divergent term and the physical terms. The Gordon identity of the Dirac current can be used, such that the finite part of (65) yields the relation

$$\Gamma_{fin}^\mu(q^2) = \gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2), \quad (66)$$

where F_1 and F_2 are the form factors, and q is defined as the photon's external momentum $q^\mu := p^\mu - p'^\mu$. We also have used the usual on-shell conditions for the fermion external momentum, i. e., $p^2 = p'^2 = m^2$. Thus, the form factors at the one-loop approximation are given by

$$\begin{aligned}
F_1(q^2) = & \frac{\alpha}{2\pi} \int_0^1 dx \int_0^{1-x} dy \\
& \frac{1 + (x + y + 1)^2 + (1 - x)(1 - y) q^2/m^2}{(x + y)^2 - xy q^2/m^2} \\
& + \frac{\alpha \chi_e^2}{2\pi} \int_0^1 dx \int_0^{1-x} dy \\
& \frac{1 + (x + y + 1)^2 + (1 - x)(1 - y) q^2/m^2}{(x + y)^2 + (1 - x - y) m_X^2/m^2 - xy q^2/m^2}, \quad (67)
\end{aligned}$$

and

$$\begin{aligned}
F_2(q^2) = & \frac{\alpha}{2\pi} \int_0^1 dx \int_0^{1-x} dy \frac{2(x + y)(1 - x - y)}{(x + y)^2 - xy q^2/m^2} \\
& + \frac{\alpha \chi_e^2}{2\pi} \int_0^1 dx \int_0^{1-x} dy \\
& \frac{2(x + y)(1 - x - y)}{(x + y)^2 + (1 - x - y) m_X^2/m^2 - xy q^2/m^2}. \quad (68)
\end{aligned}$$

The first factor F_1 is the origin of the infrared divergences in the model. When $q^2 = 0$, there is a divergence in the first integral of (67) due to integration in the Feynman's parameters, while it second integral remains finite. The second factor F_2 gives an important contribution to the electron anomalous magnetic moment. This contribution emerges when $q^2 = 0$, where the X-boson vertex brings

out the correction

$$F_2(q^2 = 0) = \frac{\alpha}{2\pi} + \frac{\alpha \chi_e^2}{2\pi} \frac{m_e^2}{m_X^2} \int_0^1 dx \times \int_0^{1-x} dy \frac{2(x+y)(1-x-y)}{1-x-y+(x+y)^2 m_e^2/m_X^2}, \quad (69)$$

here the first term is just the contribution of the ordinary QED, and the second integral is the contribution of the mass m_X . We expand the previous integral for $m_e^2/m_X^2 \ll 1$, so we obtain the result at the lower order

$$F_2(q^2 = 0) \simeq \frac{\alpha}{2\pi} \left(1 + \frac{2}{3} \chi_e^2 \frac{m_e^2}{m_X^2} \right). \quad (70)$$

If we use the known experimental uncertainty in the electron magnetic moment, it leads us to the upper bound :

$$|\chi_e| \frac{m_e}{m_X} \lesssim 6 \times 10^{-7}. \quad (71)$$

Using the values $m_e = 0.5 \text{ MeV}$ and $m_X = 17 \text{ MeV}$, we obtain the upper bound of $|\chi_e| \lesssim 2 \times 10^{-5}$. This estimate is agreement with those obtained in the recent literature.

IX. FINAL REMARKS

The recent propose of a new neutral boson explains the experimental anomalies that emerge in the 8-Beryllium nuclear decay. The solution for this puzzle implies that the mass of the X -boson must be around the $m_X = 17 \text{ MeV}$. This conjecture plays a fundamental role of a possible new physics at the MeV-scale, that so it could be the announcement of a Fifth fundamental interaction. Furthermore, the X -boson couples kinetically through the χ -mixing parameter with the usual massless photon. Other important property is the protophobic interaction with

the fermions of the Standard Model (SM). In this protophobic interaction, the coupling constants have weaker magnitude in relation to those of the SM. Thereby, the X -boson introduces an extra abelian group $U(1)_X$ to the unification of the fundamentals interaction.

We have constructed a model with the composition group $U(1)_{em} \times U(1)_X$ that can describe the unification of the new X -interaction with the quantum electrodynamics. The Higgs model was introduced to give the mass of $m_X = 17 \text{ MeV}$, that consequently, fixes a VEV-scale around the $v = 6.3 \text{ GeV}$ by the recent experimental constraints. Thus, the hidden Higgs is estimated to have a mass in the range $2.8 \text{ GeV} < m_H < 8.5 \text{ GeV}$. After the spontaneous symmetry breaking, we get a renormalizable and unitary model with electromagnetic symmetry $U(1)_{em}$.

Posteriorly, we study the protophobic interaction of the X -boson with the leptons through the elements of quantum field theory, like the decay rate in that we estimate a lifetime of order $\tau \lesssim 10^{-14} \text{ s}$. As a second application, we obtain the Yukawa potential correspondent to the electron-positron scattering (42). Following the quantum field theory, we calculate the contribution to the electron physical mass due to the protophobic interaction. The perturbative series of the X -boson full propagator was obtained, and the vacuum polarization at the one-loop gives a contribution to the previous Yukawa potential. For end, the correction to the QED vertex was calculated, and the anomalous magnetic moment $g - 2$ estimates a χ_e -parameter of the order $|\chi_e| \lesssim 2 \times 10^{-5}$. This result is agreement with the recent experimental constraints.

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